



Recursive Structures and Processes

Announcements

- **LS15: Recursive Structures** due tomorrow and 11:59pm
- **EX07: Linked List Utility Functions** released today

First, a review of recursion...

Review: Recursive function checklist:

Base case:

- ❑ Does the function have a clear base case?
 - ❑ Ensure the base case returns a result directly (without calling the function again).
- ❑ Will the base case *always* be reached?

Recursive case:

- ❑ Does the function have a recursive case that *progresses toward the base case*?
 - ❑ Does the recursive call have the right arguments? The function should call itself on a simpler or smaller version of the problem.
- ❑ Have you tested your function with multiple cases, including edge cases?

Review: Stack Overflow and Recursion Errors

When a programmer writes a function that calls itself indefinitely (*infinitely*), the **function call stack** will *overflow*...

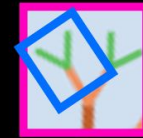
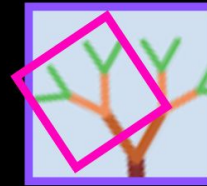
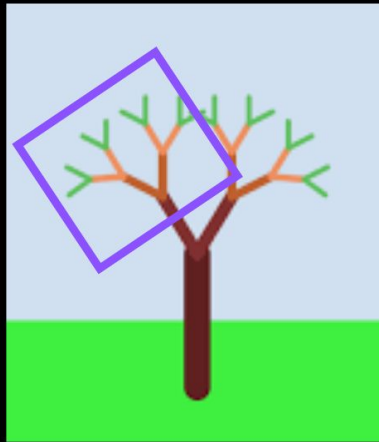
This leads to a **Stack Overflow** or **Recursion Error**:

```
RecursionError: maximum recursion depth exceeded while  
calling a Python object
```

Recursion: defining an operation/object in terms of itself

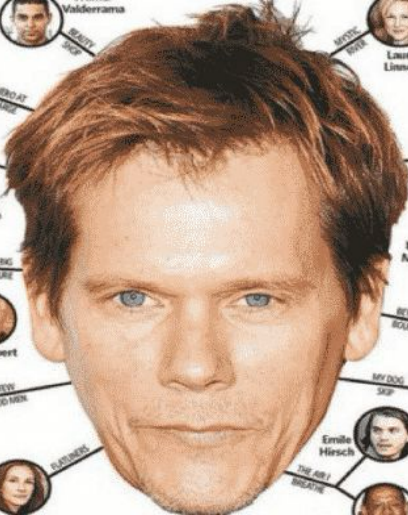
A real-world phenomenon! Examples:

- **You** have **parents**, who have **parents**, who have **parents**, who have **parents**, who...
... were **early humans**
- A **tree** has **branches**, which have **branches**, which have **branches**, which...
... have **leaves**



Six degrees of Kevin Bacon

graph/network



Coordinating plans with individual phone calls

linked list

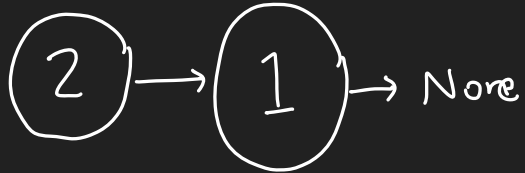
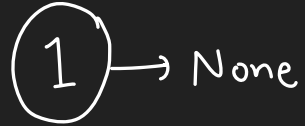
Anatomy of a Singly-Linked List

id:1

Node	
value	1
next	None

id:2

Node	
value	2
next	id:1



Memory diagram

```
1  from __future__ import annotations
2
3  class Node:
4      value: int
5      next: Node | None
6
7      def __init__(self, val: int, next: Node | None):
8          • self.value = val
9            self.next = next
10
11  # Note: There are no errors!
12  two: Node = Node(2, None)
13  one: Node = Node(1, two)
14  # We'll extend this diagram shortly, leave room
```

Note: An arrow labeled "or" points from the type annotation `Node | None` in line 5 to the `None` argument in the `__init__` call in line 12.

Output

Stack

Globals	Node
one id: 2	id: 0
	two id: 1

Node# __init__	
RA 12	self id: 1
RV id: 1	val 2
	next None

Node# __init__	
RA 13	self id: 2
	val 1
RV id: 2	next id: 1

Heap

id: 0 | class lines 3-9

id: 1 | Node

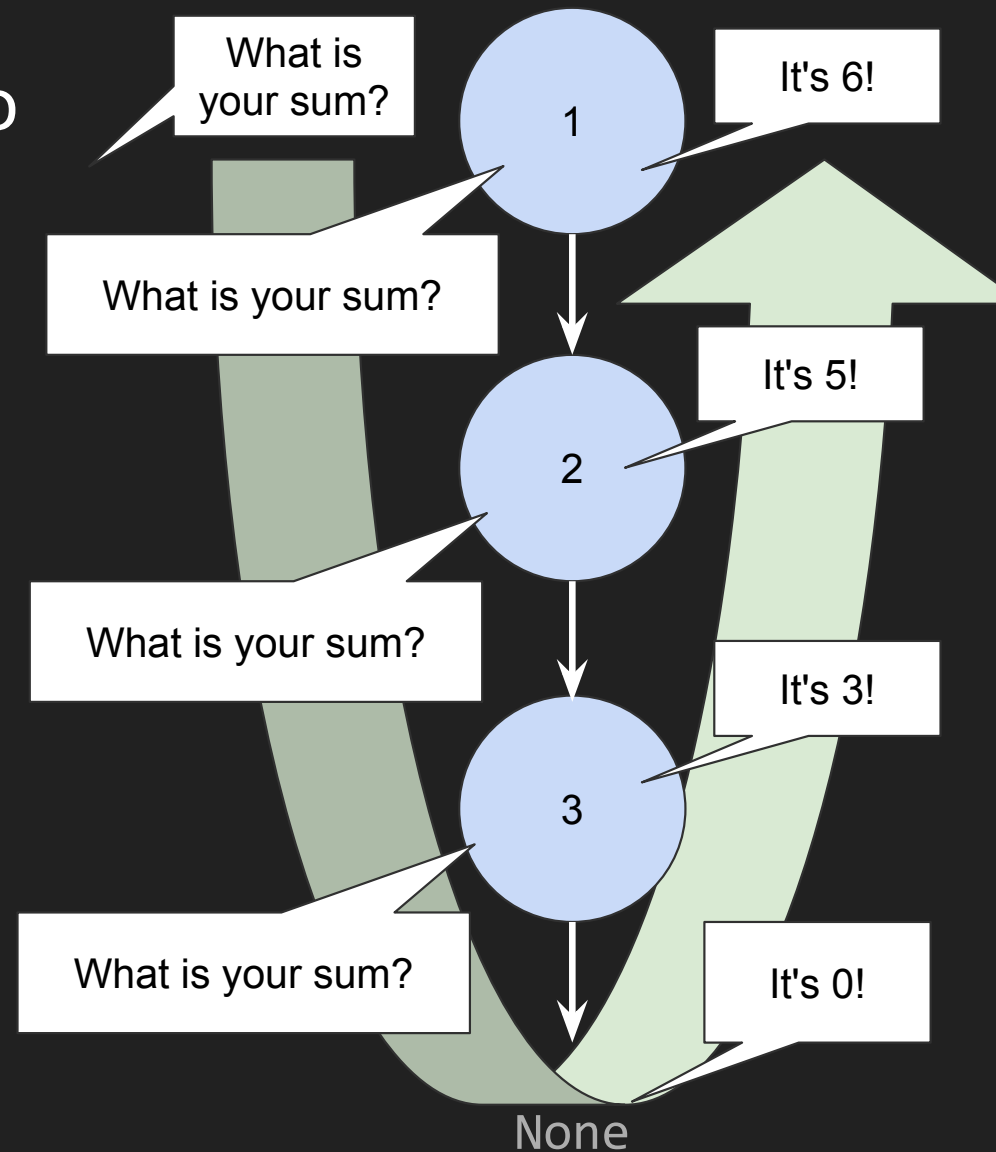
value	2
next	None

id: 2 | Node

value	1
next	id: 1

A Recursive `sum` Algorithm Demo

1. When you are asked, "what is your sum?"
2. Ask the **next** Node, "what is your sum?"
Wait patiently for an answer!
3. Once the answer is returned back to you, add **your value to it**, then turn to the person who asked you and give them this answer.



In your exercises folder, create a folder named `ex07`. In `ex07`, create a file named `linked_list.py`. Then, copy the following code into `linked_list.py`

```
1 from __future__ import annotations
2
3
4 class Node:
5     """Node in a singly-linked list recursive structure."""
6
7     value: int
8     next: Node | None
9
10    def __init__(self, value: int, next: Node | None):
11        self.value = value
12        self.next = next
13
14
15 two: Node = Node(2, None)
16 one: Node = Node(1, two)
```

Let's write a recursive function called `sum`!

```
1  from __future__ import annotations
2
3  class Node:
4      value: int
5      next: Node | None
6
7      def __init__(self, val: int, next: Node | None):
8          self.value = val
9          self.next = next
10
11  # Note: There are no errors!
12  two: Node = Node(2, None)
13  one: Node = Node(1, two)
14  # We'll extend this diagram shortly, leave room
```

Write a function called `sum` that adds up the `values` of all `Nodes` in the linked list.

For reference: recursive function checklist:

Base case:

- ❑ Does the function have a clear base case?
 - ❑ Ensure the base case returns a result directly (without calling the function again).
- ❑ Will the base case *always* be reached?

Recursive case:

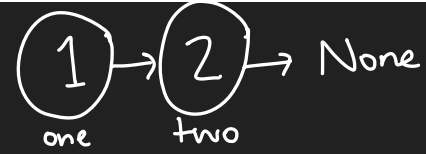
- ❑ Does the function have a recursive case that *progresses toward the base case*?
 - ❑ Does the recursive call have the right arguments? The function should call itself on a simpler or smaller version of the problem.
- ❑ Have you tested your function with multiple cases, including edge cases?

sum(head = one)

⋮
→ return head.value
return 1
return 3

+ sum(^{two}head.next)

⋮
→ return head.value
return 2
return 2

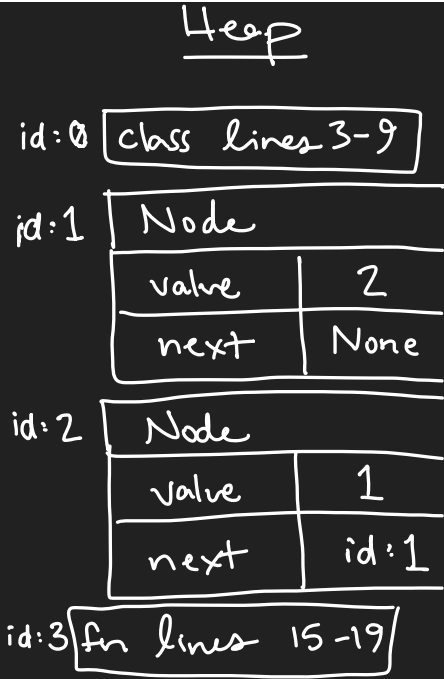
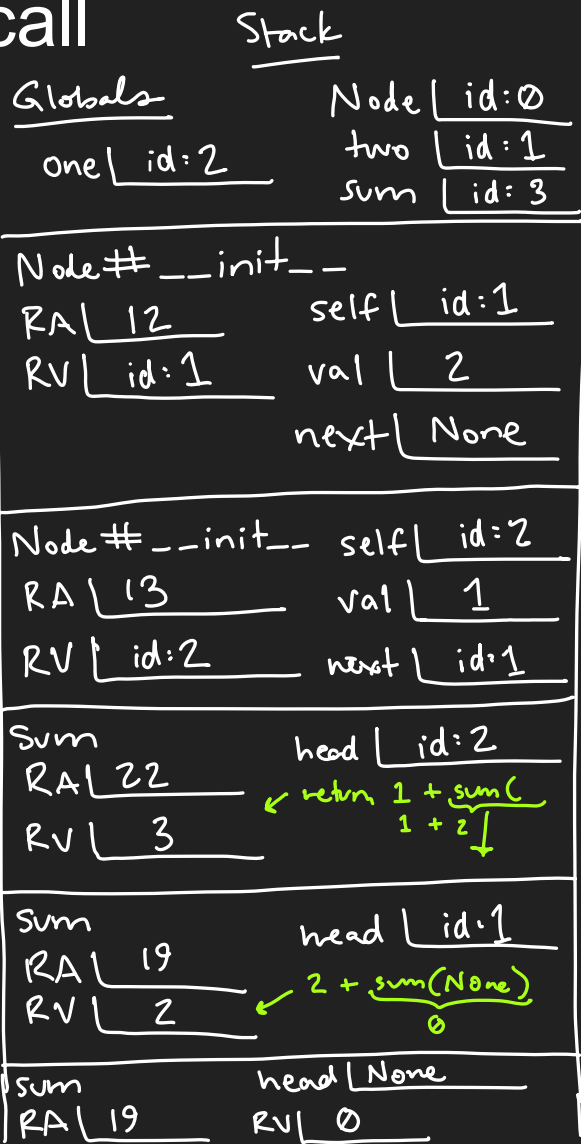


+ sum(^{None}head.next)
+ 0
⋮
→ return 0
(base case)

Diagramming the sum function call

```
1  from __future__ import annotations
2
3  class Node:
4      value: int
5      next: Node | None
6
7      def __init__(self, value: int, next: Node | None):
8          self.value = value
9          self.next = next
10
11
12  two: Node = Node(2, None)
13  one: Node = Node(1, two)
14
15  def sum(head: Node | None) -> int:
16      if head is None:
17          return 0
18      else:
19          return head.value + sum(head.next)
20
21
22 • print(3sum(one))
```

Output
3



More practice!

A Recursive `last` Algorithm Demo

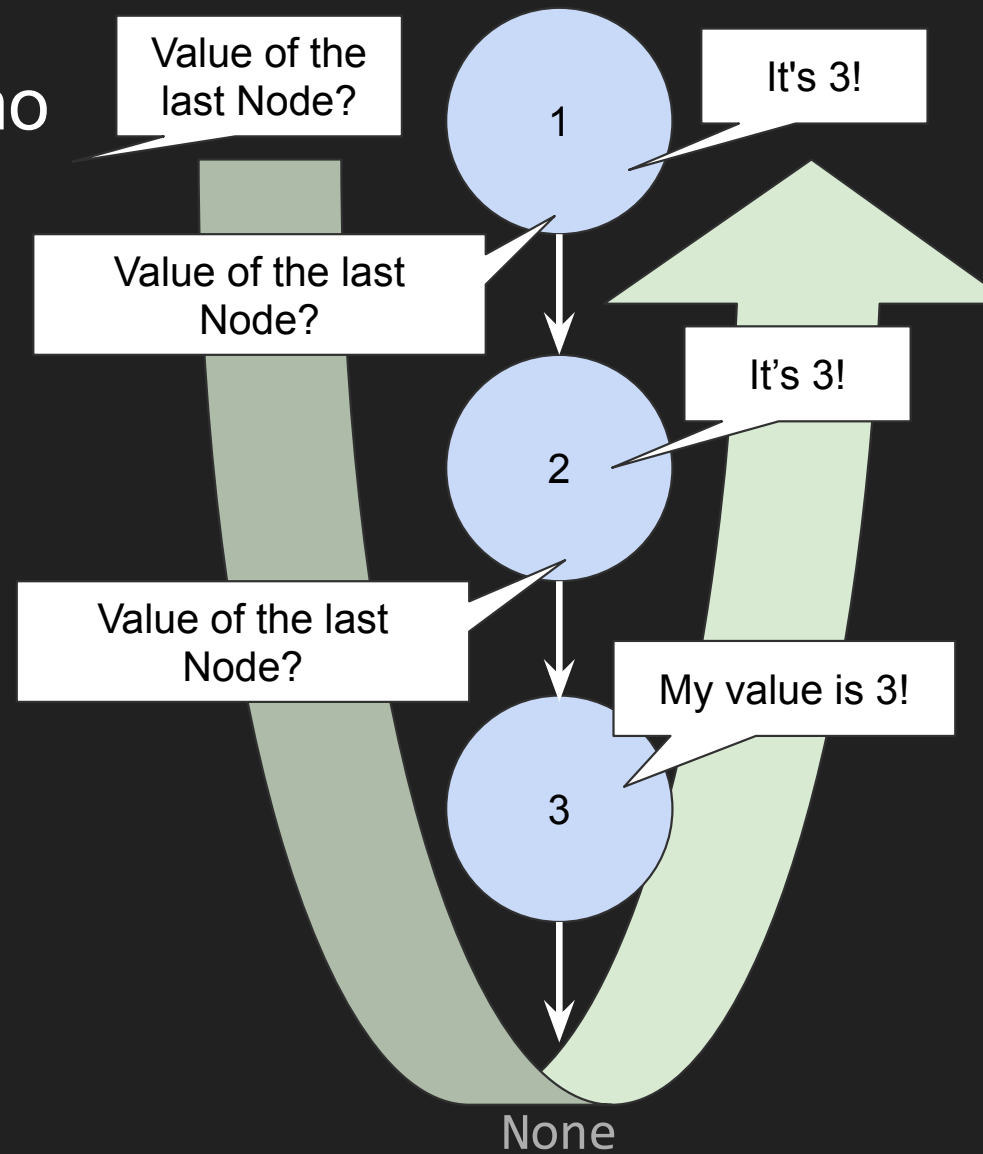
1. When you are asked,
"What is the value of the last Node?"

If you're ***not the last Node***:

2. Ask the ***next*** Node,
"What is the value of the last Node?"
Wait patiently for an answer!
3. Once the answer is returned back to you,
turn to the person who asked you and
give them this answer.

If you ***are the last Node***:

2. Tell them, "my value is ____!" and share your
value.



Let's write the `last` function in VS Code!  

insert_after Algorithm Demo

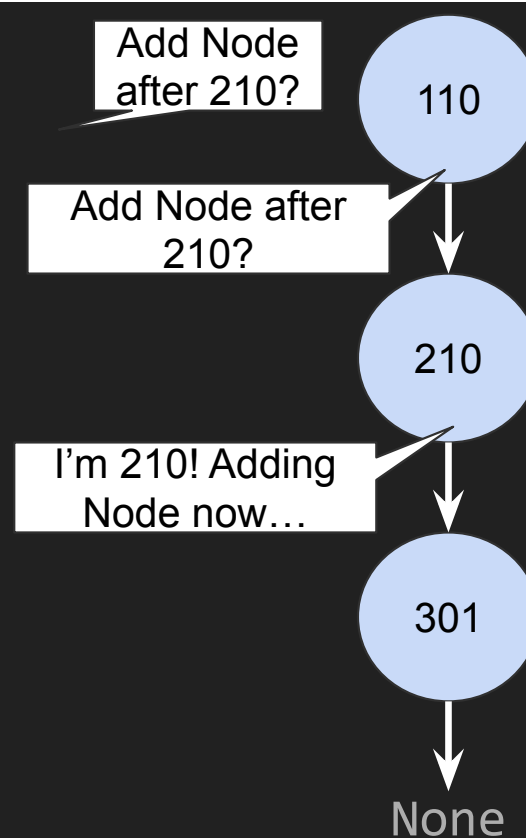
1. When you are asked,
"Can you add a Node with a value of 211 after the
Node with value 210?"

If your value *is not 210*:

2. Ask the next Node,
"Can you add a Node with a value of 211 after the
Node with value 210?"
Wait patiently for an answer!
3. Once the answer is returned back to you, turn to
the person who asked you and give them this
answer.

If your value *is 210*:

2. Invite a new friend to the list! You will now point to
them, and they will point to the person you were
previously pointing to. New Node, you'll say "I was
added!!"



insert_after Algorithm Demo

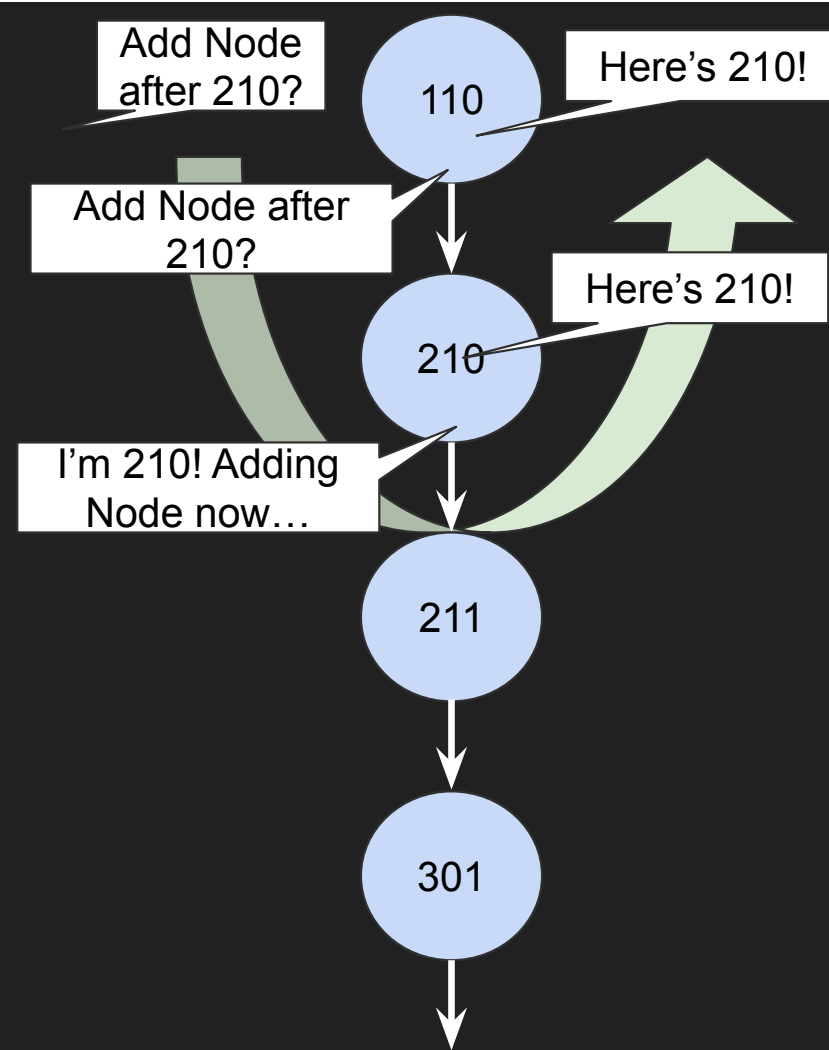
1. When you are asked,
"Can you add a Node with a value of 211 after the
Node with value 210?"

If your value **is not 210**:

2. Ask the next Node,
"Can you add a Node with a value of 211 after the
Node with value 210?"
Wait patiently for an answer!
3. Once the answer is returned back to you, turn to
the person who asked you and give them this
answer.

If your value **is 210**:

2. Invite a new friend to the list! You will now point to
them, and they will point to the person you were
previously pointing to. New Node, you'll say "I was
added!!"



Let's write pseudocode for the `insert_after` function

Let's write the `insert_after` function in VS Code!

